CHAPTER 1

TIMSS 2023 Mathematics Framework

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Overview

All children can benefit from developing an understanding of mathematics and facility with the mathematics needed in today's technological world. Mathematics is essential in daily life as well as in many career fields such as engineering, architecture, accounting, banking, business, medicine, ecology, and aerospace. Mathematics is vital to economics and finance, as well as to computing technology and software development. The ability to learn new skills and to solve problems is paramount in today's changing world.

This chapter presents the assessment frameworks for the two TIMSS 2023 mathematics assessments:

- TIMSS Mathematics—Fourth Grade
- TIMSS Mathematics—Eighth Grade

As described in the Introduction, the *TIMSS 2023 Mathematics Framework* for the fourth and eighth grades builds on TIMSS's 28-year history of assessments every four years since 1995, with this being the eighth assessment in the series.

In general, the fourth and eighth grade frameworks are similar to those used in TIMSS 2019. Minor updates reflect the curricula, standards, and frameworks of the participating countries as reported in the *TIMSS 2019 Encyclopedia*[1] and suggestions from the TIMSS 2023 National Research Coordinators.

TIMSS 2023 is Fully Digital

The transition to digital assessment initiated in about half the TIMSS 2019 countries will be completed in TIMSS 2023, such that TIMSS 2023 is fully digital.

An overarching goal for TIMSS 2023 is capitalizing on the benefits of computer-based assessments, including a fully digital item development system to incorporate new and better assessment items and methods, and knowledge of the improved capacity for innovative item development influenced the present framework. Digital assessment allows:

- Accounting for dynamic aspects of mathematical concepts such as relationships and geometric operations by offering interactive simulations or tools closer to what these concepts really address.

- Improved assessment of the reasoning cognitive processes by relegating some procedural and secondary calculation tasks to the computer, allowing students to focus on strategy and mathematical thinking.
- Process data associated with students' response patterns that can be used to learn more about students' problem solving strategies, misconceptions, and approaches to test taking.
- Enrichment of the overall testing display and response formats, helping to improve students' engagement and motivation to participate in TIMSS.

Expectations for a Range of Problem Solving Contexts

Previous TIMSS Mathematics frameworks have not been clear about the degree of emphasis that should be placed on solving problems in context. Typically, solving problems was included as part of at least one topic within a content domain, implying that some items within a content domain were expected to be situated in contexts. Also, all items in the assessment are classified by cognitive domain— knowing, applying, or reasoning— with 60 to 65 percent of the items requiring applying and reasoning in a problem solving context.

The *TIMSS 2023 Mathematics Framework* specifies that approximately 85 percent of the items covering the topics in each content domain should be situated in a problem solving context. This is consistent with other aspects of the framework, and clarifies that problem solving is an overarching goal of TIMSS Mathematics, and not associated only with particular topics. The contexts can range from straightforward to complex extended scenarios, such as in the Problem Solving and Inquiry Tasks (PSIs). It is important, however, that at least 15 percent of the items are presented without context to be able to examine the possible effects of reading load.

Organization of TIMSS 2023 Mathematics Framework

The mathematics assessment framework for TIMSS 2023 is organized around two dimensions:

- Content dimension, specifying the subject matter domains to be assessed
- Cognitive dimension, specifying the thinking processes to be assessed

Exhibit 1.1 shows the target percentage of testing score points devoted to each content and cognitive domain for the TIMSS 2023 fourth and eighth grade assessments.

Exhibit 1.1: Target Percentages of the TIMSS 2023 Mathematics Assessment Devoted to Content and Cognitive Domains at the Fourth and Eighth Grades

Fourth Grade

Content Domains	Percentages
Number	50%
Measurement and Geometry	30%
Data	20%

Eighth Grade

Content Domains	Percentages
Number	30%
Algebra	30%
Geometry and Measurement	20%
Data and Probability	20%

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

The content domains differ for the fourth and eighth grades, reflecting the mathematics widely taught at each grade. There is more emphasis on number at the fourth grade than at the eighth grade. Algebra becomes a topic of its own in eighth grade, whereas the introductory algebraic topics assessed at the fourth grade are included in the number topic area. The eighth grade geometry domain includes measurement but also a deeper inclusion of purely geometric topics. The fourth grade data domain focuses on reading, representing, and interpreting data, whereas at the eighth grade it includes more emphasis on drawing conclusions from data, basic statistics, and the fundamentals of probability.

It is important to highlight that TIMSS assesses a range of situations within mathematics, with well over half (60-65%) the items requiring students to use applying and reasoning skills. The cognitive domains are the same for both grades, but with less emphasis in the eighth grade on the knowing domain and greater emphasis on the reasoning domain.

Following this brief introduction, the chapter begins with the fourth grade content domains, identifying the three main content domains and the assessment topics within each domain. Chapter 1 continues with the description of the eighth grade domains and calculator policy. The chapter ends with the description of the cognitive domains for both grades.

Mathematics Content Domains—Fourth Grade

Exhibit 1.2 shows the TIMSS 2023 Mathematics—Fourth Grade content domains and the target percentages of assessment score points devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the fourth grade mathematics assessment, each topic receives approximately equal weight.

Exhibit 1.2: Target Percentages of the TIMSS 2023 Mathematics Assessment Devoted to Content Domains at the Fourth Grade

Fourth Grade Content Domains	Percentages
Number	50%
Measurement and Geometry	30%
Data	20%

Each of the following topics within each content area can be assessed by items measuring the knowing, applying, or reasoning cognitive domains as appropriate. Also, the items covering the topics in a content domain are expected to be situated in a range of contexts. At least 15 percent should be presented without context, and the remaining should range from straightforward problem solving situations to the complex extended scenarios in the PSIs.

Number

Number provides the foundation of mathematics in primary school. The number content domain consists of three topic areas. The 50 percent of the assessment devoted to number is apportioned as follows:

- Whole numbers (25%)
- Expressions, simple equations, and relationships (15%)
- Fractions and decimals (10%)

Whole numbers are the predominant component of the number domain and students should be able to compute with whole numbers of reasonable size. Introductory algebraic concepts also are part of the TIMSS assessment at the fourth grade, including understanding the use of variables (unknowns) in simple equations and initial understandings of relationships between quantities. However, because objects and quantities often do not come in whole numbers, it is also important for students to understand fractions and decimals. Students should be able to compare, add, and subtract familiar fractions and decimals.

Whole Numbers

1. Recognize place value of numbers to 6 digits, connect representations of numbers (words, symbols, and models including number lines), and compare numbers.
2. Add and subtract up to 4-digit numbers.

3. Multiply (up to 3-digit by 1-digit and 2-digit by 2-digit numbers) and divide (up to 3-digit by 1-digit numbers).
4. Solve problems involving odd and even numbers, multiples and factors of numbers, rounding numbers (up to the nearest powers of 10), and making estimates.
5. Combine two or more properties of numbers or operations to solve a problem.

Expressions, Equations, and Relationships

1. Find the missing number or operation in a number sentence (e.g., 17 + w = 29).
2. Match or write expressions or number sentences to represent problem situations that may involve unknowns.
3. Match, describe, or use relationships in a well-defined pattern (e.g., describe the relationship between adjacent terms and generate pairs of whole numbers given a rule).

Fractions and Decimals

1. Describe a fraction as part of a whole or collection; connect different representations of fractions (words, numbers, and models); compare the size of fractions; add and subtract simple fractions with like denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
2. Connect different representations of decimals (words, numbers, and models); compare and order decimals and relate decimals to fractions; round decimals; add and subtract decimals (up to two decimal places).

Measurement and Geometry

We are surrounded by objects of different shapes and sizes, and geometry helps us visualize and understand the relationships between shapes and sizes. Measurement is the process of quantifying attributes of objects and phenomena (e.g., length and time).

The two topic areas in measurement and geometry are as follows:

- Measurement (15%)
- Geometry (15%)

At the fourth grade, students should be able to use a ruler to measure length; perform calculations involving length, mass, volume, and time; calculate areas of shapes based on rectangles; calculate perimeters of polygons; and use cubes to determine volumes. Students should be able to identify the properties and characteristics of lines, angles, and a variety of two- and three-dimensional shapes. Spatial sense is integral to the study of geometry, and students will be asked to describe and draw a variety of geometric figures. They also should be able to analyze geometric relationships and use these relationships to draw conclusions about geometric objects.

Measurement

1. Measure, estimate, add, and subtract lengths (millimeters, centimeters, meters, kilometers).
2. Add and subtract mass (gram and kilogram), volume (milliliter and liter), and time (minutes and hours); select appropriate types and sizes of units and read scales.
3. Determine perimeters of polygons, areas of rectangles, areas of shapes covered with squares or partial squares, and volumes filled with cubes.

Geometry

1. Recognize and draw parallel and perpendicular lines, right angles, and angles smaller or larger than a right angle; compare the relative size of angles.
2. Use elementary properties, including line and rotational symmetry to describe and create common two-dimensional shapes (circle, triangles, quadrilaterals, and other polygons).
3. Use elementary properties to describe three dimensional shapes (cubes, rectangular solids, cones, cylinders, and spheres), the differences among them, and how they relate to their two-dimensional representations.

Data

The explosion of data in today's information society has resulted in a variety of visual displays of quantitative information. Often the internet, newspapers, magazines, textbooks, reference books, and articles have data represented in charts, tables, and graphs. Students need to understand that graphs and charts help organize information or categories and provide a way to compare data.

The data content domain consists of two topic areas:

- Reading and displaying data (10%)
- Interpreting, combining, and comparing data (10%)

At the fourth grade, students should be able to read and create data displays. They should be able to make inferences from data displays and use data from one or more sources to answer questions of interest.

Reading and Displaying Data

1. Read data from tables, pictographs, bar graphs, line graphs, and pie charts.
2. Create or complete tables, pictographs, bar graphs, line graphs, and pie charts.

Interpreting, Combining, and Comparing Data

1. Interpret data and use it to answer questions that go beyond directly reading data displays.
2. Combine or compare data from two or more sources, and draw conclusions based on two or more data sets.

Mathematics Content Domains—Eighth Grade

Exhibit 1.3 shows the TIMSS 2023 Mathematics—Eighth Grade content domains and the target percentages of assessment score points devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the eighth grade mathematics assessment, each topic receives approximately equal weight.

Exhibit 1.3: Target Percentages of the TIMSS 2023 Mathematics Assessment Devoted to Content Domains at the Eighth Grade

Eighth Grade Content Domains	Percentages
Number	30%
Algebra	30%
Geometry and Measurement	20%
Data and Probability	20%

As with fourth grade mathematics, each of the following topics within each content area at the eighth grade can be assessed by items measuring the knowing, applying, or reasoning cognitive domains as appropriate. Also, the items covering the topics in a content domain are expected to be situated in a range of contexts. At least 15 percent should be presented without context, and the remaining should range from straightforward problem solving situations to the complex extended scenarios in the PSIs.

Number

At the eighth grade, the 30 percent of the assessment devoted to number consists of three topic areas:

- Integers (10%)
- Fractions and decimals (10%)
- Proportions, ratios, and percentages (10%)

Building on the number content domain at the fourth grade, eighth grade students should have developed proficiency with more advanced whole number concepts and procedures as well as extended their mathematical understanding of rational numbers (integers, fractions, and decimals). Students also should understand and be able to compute with integers. Fractions and decimals are an important part of daily life and being able to compute with them requires an understanding of the quantities the symbols represent. A single rational number can be represented with many different written symbols, and students need to be able to recognize the distinctions among interpretations of rational numbers, convert between them, and reason with them. Students should be able to apply ratios, proportions, and percentages to whole number amounts.

Integers

1. Recognize and use properties of numbers and operations; find and use multiples and factors, recognize prime numbers, evaluate positive integer powers of number, and square roots of whole numbers.
2. Add and subtract positive and negative numbers, including through movement and position on a number line or using various models (e.g., thermometers, losses and gains).

Fractions and Decimals

1. Using various models and representations, compare and order fractions and decimals, and identify equivalent fractions and decimals.
2. Add, subtract, and multiply with fractions and decimals, and divide fractions and decimals by a whole number.

Proportions, Ratios, and Percentages

1. Determine proportions and ratios of quantities (e.g. rates, scales on maps).
2. Apply or find percentages; convert between percentages and fractions or decimals.

Algebra

The 30 percent of the assessment devoted to algebra is comprised of two topic areas:

- Expressions, operations, and equations (20%)
- Relationships and functions (10%)

Patterns and relationships are pervasive in the world around us. Students should be able to use algebraic models and express relationships algebraically. They need to be able to rearrange formulas and substitute values into formulas. Their conceptual understanding can extend to linear equations for calculations about quantities that change at constant rates. Linear and simple non-linear functions can be used to describe what will happen to a variable when a related variable changes.

Expressions, Operations, and Equations

1. Find the value of an expression or a formula given values of the variables.
2. Simplify algebraic expressions involving sums, products, differences, and positive integer powers; compare expressions to decide if they are equivalent.
3. Write expressions, equations, or inequalities to represent problem situations.
4. Solve linear equations, linear inequalities, and simultaneous linear equations in two variables, including validating values as solutions.

Relationships and Functions

1. Interpret, relate, and generate representations of linear functions in tables, graphs, or words; recognize properties of linear functions including slope and intercepts.

2. Interpret, relate, and generate representations of simple non-linear functions (e.g., quadratic) in tables, graphs, or words; generalize linear and non-linear pattern relationships or sequences, using words, or algebraic expressions.

Geometry and Measurement

The geometry and measurement content domain at the eighth grade consists of one topic area:

- Geometry and Measurement (20%)

Extending the understanding of shapes and measures assessed at the fourth grade, eighth grade students should be able to analyze the properties of a variety of two- and three-dimensional figures and calculate perimeters, areas, and volumes. They should be able to provide explanations based on geometric relationships, such as congruence, similarity, and the Pythagorean Theorem.

Geometry and Measurement

1. Recognize and draw types of angles and pairs of lines and use the relationships between angles on lines and in geometric figures, including those involving the measures of angles and line segments; read and plot points in the Cartesian plane.
2. Recognize two-dimensional shapes and use their geometric properties (e.g. sums of interior angles of triangles and quadrilaterals, properties of isosceles triangles), including to calculate length and area, and use the Pythagorean Theorem.

   Note: Two-dimensional shapes include circles; scalene, isosceles, equilateral, and right-angled triangles; trapezoids, parallelograms, rectangles, rhombuses, and other quadrilaterals; as well as other polygons including pentagons, hexagons, octagons, and decagons.

3. Determine the results of geometric transformations (translations, reflections, and rotations) in the plane; recognize and use properties of congruent and similar triangles and rectangles.
4. Recognize three-dimensional shapes and use their properties to calculate surface area and volume; relate three-dimensional shapes with their two-dimensional representations.

   Note: Three-dimensional shapes include prisms, pyramids, cones, cylinders, and spheres.

Data and Probability

The data and probability content domain contains two topic areas:

- Data (15%)
- Probability (5%)

Increasingly, the more traditional forms of data display (e.g., bar graphs, line graphs, pie graphs, pictographs) are being supplemented by an array of new graphic forms (e.g., infographics). By the eighth grade, students should to be able to read and extract the important meaning from a variety of visual displays. It is also important for eighth grade students to be familiar with the statistics underlying data distributions and how these relate to the shape of data graphs. Students should know how to organize and represent data. Students also should have an initial grasp of some concepts related to probability.

Data

1. Interpret data from one or more sources (e.g., interpolate and extrapolate, make comparisons, draw conclusions).
2. Organize and represent data to help answer questions. Representations include all those at fourth grade (tables, pictographs, bar graphs, line graphs, and pie charts) and in addition, histograms, dot plots, scatter plots, clustered and stacked bar charts, and infographics.
3. Summarize data distributions; calculate, use, or interpret mean and median; recognize the effect of spread and outliers.

Probability

1. For simple and compound events: determine theoretical probability (based on proportions of favorable outcomes, e.g., rolling a fair die or drawing marbles of a particular color from a bag); estimate empirical probability (based on experimental outcomes).

Calculator Use at the Eighth Grade

At the eighth grade, students will be permitted to use the TIMSS on-screen calculator. This calculator has the four basic functions (+, −, ×, ÷), a square root key, and the negative sign. Students will not be permitted to bring their own calculators. On the whole the mathematics items are developed to be calculator neutral and do not advantage or disadvantage students whether or not they use calculators. A notable exception is the (very few) items that require the taking of a square root.

Mathematics Cognitive Domains—Fourth and Eighth Grades

In order to respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. These include the ability to select and carry out procedures, apply knowledge to solve problems, make logical deductions, and give reasons for an assertion. Describing these skills plays a crucial role in the development of an assessment like TIMSS 2023, ensuring that the survey covers the appropriate range of cognitive skills across the content domains already outlined.

The first domain, *knowing*, covers the facts, concepts, and procedures students need to know, while the second, *applying*, focuses on the ability of students to apply knowledge and conceptual understanding in a range of situations. The third domain, *reasoning*, involves the logical, systematic thinking that students need to use to generate and justify solutions to problems, make inferences, and deal with complex relationships between mathematical objects.

Knowing, applying, and reasoning are exercised in varying degrees when students display their mathematical competency, which goes beyond content knowledge. These TIMSS cognitive domains encompass the competencies of providing a mathematical argument to support a strategy or solution,

representing a situation mathematically (e.g., using symbols and graphs), creating mathematical models of a problem situation, and using tools such as a ruler or a calculator.

The three cognitive domains are used for both grades, with each item categorized into one of the three domains. Reflecting the difference in age and experience of students, the balance of score points differs between fourth and eighth grade (see Exhibit 1.4). For both grades, each content domain will include some items developed to address each of the three cognitive domains. For example, the number domain will include knowing, applying, and reasoning items as will the other content domains.

Exhibit 1.4 shows the target percentages of score points devoted to each cognitive domain for the fourth and eighth grade assessments.

Exhibit 1.4: Target Percentages of the TIMSS 2023 Mathematics Assessment Devoted to Cognitive Domains at the Fourth and Eighth Grades

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

The following sections describe the types of cognitive skills particular to each of the three cognitive domains. Items are classified according to cognitive skills to ensure a range of coverage within each cognitive domain. However, there are no specified targets for the percentages of score points for each cognitive skill.

Knowing

Facility in applying mathematics, or reasoning about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she understands, the greater the potential for engaging with a wide range of problem situations.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the knowledge that provides the basic language of mathematics, as well as the essential mathematical concepts and properties that form the foundation for mathematical thought.

Procedures form the foundation of the mathematics needed for solving problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools in relatively familiar and routine tasks. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., a × b = ab, a + a + a = 3a).
Identify	Identify numbers, expressions, quantities, and shapes. Recognize when entities are mathematically equivalent. Read information from graphs, tables, texts, or other sources.
Order	Order and classify numbers, expressions, quantities, and shapes by common properties.
Compute	Compute arithmetic operations with whole numbers, fractions, decimals, and integers using algorithmic procedures. Carry out straightforward algebraic manipulation.

Applying

The applying domain involves the application of mathematics in a range of situations. Problem solving is central to this domain. Students will need to select suitable operations, strategies, and tools for solving problems. Many of the problems are set in real life situations, requiring students to formulate the problem in mathematical terms before implementing a solution. In these problems, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create representations is fundamental to success in the subject.

Other problems may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. With these problems, a mathematical representation might be given and students might need to interpret the representation or generate an equivalent representation in order to solve the problem.

Formulate	Determine efficient/appropriate operations, strategies, and tools for solving problems.
Implement	Implement suitable strategies and operations to produce solutions to problems.
Represent	Represent data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.

Reasoning

Reasoning mathematically involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems. Evidence of reasoning processes can be found in the explaining or justifying of a solution method, or the making of valid inferences on the basis of information and evidence. Reasoning is required in analyzing or generalizing mathematical relationships.

Even though many of the cognitive skills listed in the reasoning domain may be drawn on when thinking about and solving complex problems, each by itself represents a valuable outcome of

mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

Analyze	Analyze, describe, or use relationships among numbers, expressions, quantities, and shapes.
Integrate	Link different elements of knowledge, related representations, and procedures.
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy or solution.

References

1 Kelly, D.L., Centurino, V., Martin, M.o., & Mullis, I.V. S. (Eds.) (2020). *TIMSS 2019 encyclopedia: Education policy and curriculum in mathematics and science*. Retrieved from Boston College, TIMSS & PIRLS International Student Center website: http://timssandpirls.bc.edu/timss2019/encyclopedia/