# APPENDIX B

Example Advanced Mathematics Items

Two mathematical models are proposed to predict the return y, in dollars, from the sale of x thousand units of an article (where $0 < x < 5$). Each of these models, P and Q, is based on different marketing methods.

$$\text{model P:} \qquad y = 6x - x^2$$
$$\text{model Q:} \qquad y = 2x$$

For what values of x does model Q predict a greater return than model P?

Ⓐ $0 < x < 4$

Ⓑ $0 < x < 5$

Ⓒ $3 < x < 5$

Ⓓ $3 < x < 4$

● $4 < x < 5$

MA13003
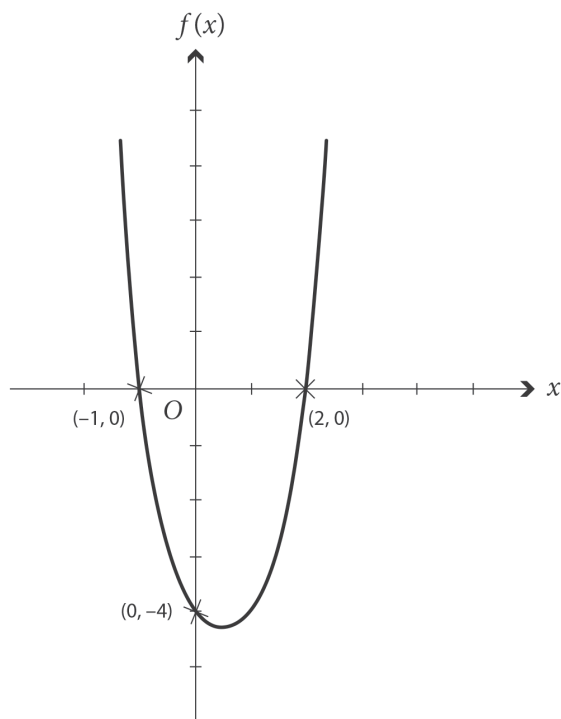
The graph of the function f is shown above. The equation of the function f is given by $f(x) = ax^2 + bx + c$. Find the values of a, b, and c.
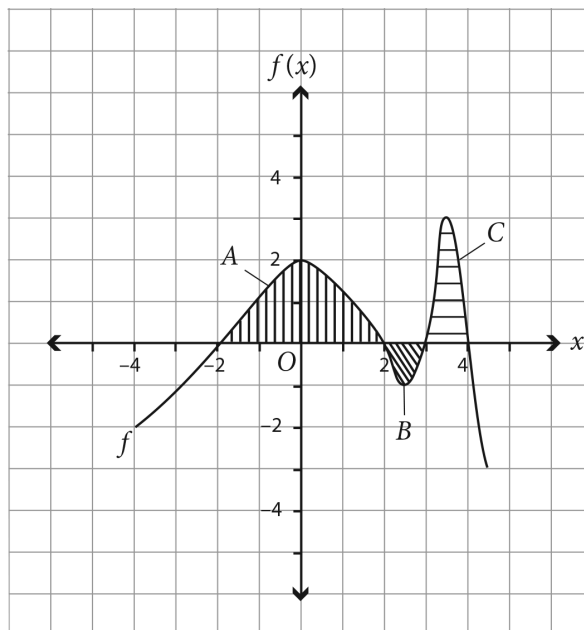
Show your work.

$0 = a(-1)^2 + b(-1) + c$

$-4 = a(0)^2 + b(0) + c \quad \Rightarrow \quad \boxed{c = -4}$

$0 = a(2)^2 + b(2) + c$

$0 = a - b - 4 \quad \rightarrow \quad a = 4 + b$

$0 = 4a + 2b - 4$

$\qquad\qquad\qquad\qquad \searrow \boxed{a = 2}$

$0 = 4(4 + b) + 2b - 4$

$0 = 12 + 6b$

$\boxed{b = -2}$ $\qquad f(x) = 2x^2 - 2x - 4$

MA23141

TIMSS & PIRLS
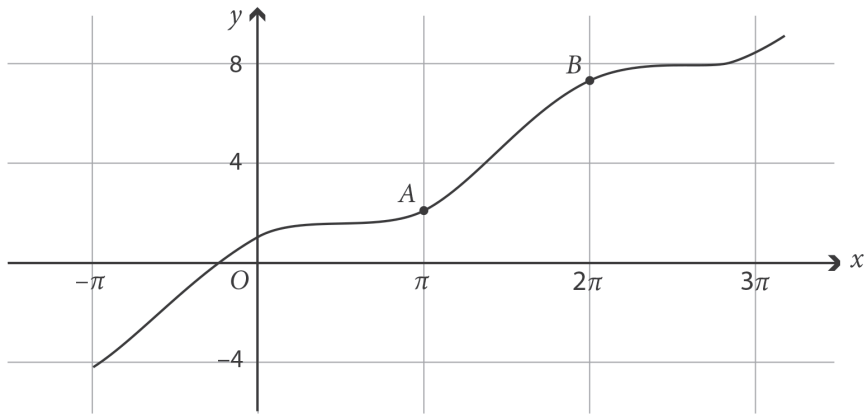International Study Center
Lynch School of Education, Boston College

For the areas between the graph of $f(x)$ and the x-axis shown above, area $A = 4.8$ units, area $B = 0.8$ units, and area $C = 2$ units.

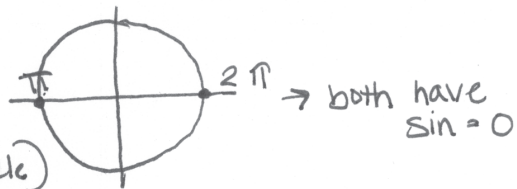What is the value of the definite integral $\int_{-2}^{4} f(x)dx$?

Ⓐ 5.6

⬤ 6.0

Ⓒ 6.8

Ⓓ 7.6

Sophia is studying the graph of the function $y = x + \cos x$ shown above. She says that the slope at point A is the same as the slope at point B. Explain why she is correct.
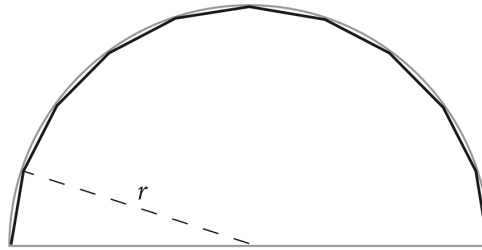
If $f = x + \cos x$
then $f' = 1 - \sin x$

A both π and 2π, the
sine (y value on unit circle)
is 0.

π 2π → both have
                    sin = 0

At both π and 2π, $f' = 1$. So f has the same
slope at $x = \pi$ and $x = 2\pi$

TIMSS & PIRLS
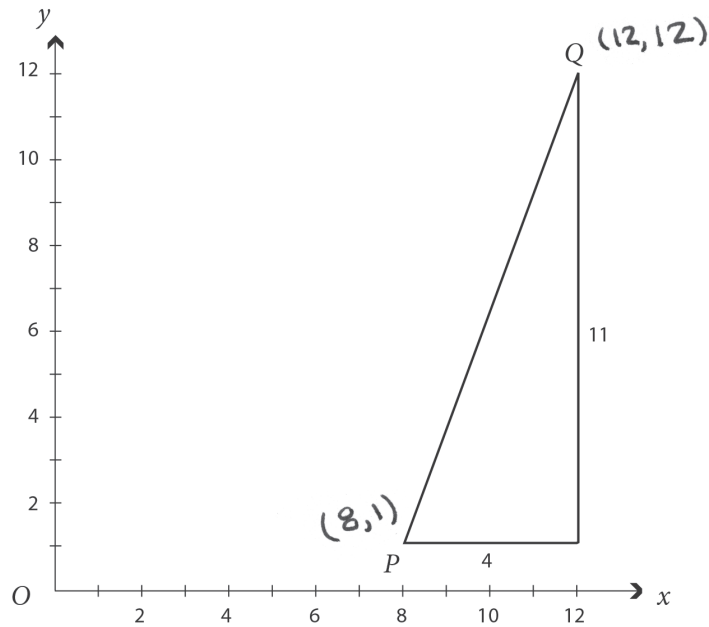International Study Center
Lynch School of Education, Boston College

The figure shows a semicircular room seen from above. An architect is placing 10 flat windows in the room as shown. If the radius of the circle is r, which of the following equations would allow the architect to determine the width of each window?

Ⓐ $w = r \sin 9°$

● $w = 2r \sin 9°$

Ⓒ $w = r \cos 18°$

Ⓓ $w = 2r \sin 18°$

MA23021

A straight line *l* passes through the points *A* (1,–2) and *B* (3, 4).
Is the line *l* parallel with *PQ*?

No

Give a reason to support your answer.

Parallel lines have the same slopes.

Slope PQ = $\frac{12-1}{12-8}$ = $\frac{11}{4}$

Slope AB = $\frac{4-(-2)}{3-1}$ = $\frac{6}{2}$ = 3

AB ∦ PQ.

TIMSS & PIRLS
International Study Center
Lynch School of Education, Boston College